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ABSTRACT

The field test of the Developing Mathematical Processes (DMP) program was conducted jointly by the Falconer Central School, St. Mary's Elementary School in Dunkirk, New York, and the Teacher Education Research Center at the State University College in Fredonia, New York. DMP is a research-based, innovative, process-oriented elementary mathematics program developed at the Research and Development Center for Cognitive Learning at the University of Wisconsin-Madison. After two months in the field test, participating teachers were asked to evaluate the program with open-ended comment. Most responded favorably, some reserved judgment, and some were having a little trouble adjusting to the new program. At the end of the year, however, all 16 teachers responding to an attitude survey reported favorable reactions to the program. On criterion-referenced measures, the second-grade DMP classes gained substantially over the control group, but the first grades gained equally. On standardized tests, the control group scored slightly higher on computations and mathematical concepts. Inasmuch as DMP is sequenced along a different continuum, the standardized achievement test scores represent an unfair and invalid measure of mathematical success as taught through DMP. (HMD)

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DEVELOPING MATHEMATICAL PROCESSES (DMP)

FIELD TEST EVALUATION, 1972-73

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Nutty Nimrod

Foreword

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The Developing Mathematical Processes (DMP) program field test was supported jointly by the Falconer Central School; the St. Mary's Elementary School, Dunkirk; and the Teacher Education Research Center, State University College, Fredonia, New York.

DMP is a research-based, innovative, process-oriented elementary mathematics program that was developed at the Research and Development Center for Cognitive Learning, University of Wisconsin--Madison.

The project has fulfilled the following purposes: (1) for the schools involved, it has provided a change thrust to upgrade elementary mathematics instruction for children; (2) for the Teacher Education Research Center, it has provided another vehicle for the support of individualized instruction in collaboration with area schools; and (3) for the elementary mathematics program at the College at Fredonia, it has provided innovative mathematics materials and processes that can be offered to both pre- and in-service educators.

We appreciate the assistance of Elizabeth Alday, Falconer Central School, and Sister Jane Marie, St. Mary's School, in providing on-site leadership in getting DMP initiated in their respective schools.

Special thanks are due Doris Hall and Trudy Reep for their assistance in preparing this report.

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The cover of this report was designed by Helen McKee, Teacher Education Research Center.

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Supervising Principal
Falconer Central School

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Principal
St. Mary's Elementary School

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School Personnel

Falconer Central School Personnel

James H. Gassman	Supervising Principal
Elizabeth Alday	Elementary Coordinator
Herbert Carlson	Principal--Fenner Elementary School
Rudolf Donn	Principal--Ellington Elementary
Donald Lazarony	Principal--Kennedy Elementary
Lucy Mula	Principal--North Side Elementary School
Richard Pond	Principal--South Side Elementary School
Marian Palmer	Kindergarten Teacher--Fenner
Joan Rine	Kindergarten Teacher--North Side
Eleanor Fitzpatrick	Kindergarten Teacher--South Side
Lila McKee	Kindergarten Teacher--Kennedy Elementary
Berdina Bolling	First Grade Teacher--North Side
Helen Baldwin	First Grade Teacher--Fenner
Helen Condon	First Grade Teacher--Kennedy
Marlene DeSantis	First Grade Teacher--Fenner
Elizabeth Matthews	First Grade Teacher--Ellington
Doris Patterson	First Grade Teacher--South Side
Christine Watt	Second Grade Teacher--North Side
Amy Writeman	Second Grade Teacher--Fenner
Ruth Davis	Second Grade Teacher--Kennedy
June Bunge	Second Grade Teacher--Fenner
Carol Stanton	Second Grade Teacher--Ellington
Elizabeth Morgenstern	Second Grade Teacher--South Side

St. Mary's Elementary School Personnel

Sister Jane Marie	Principal
Helen Szczerbacki	Kindergarten Teacher
Mary Kirst	First Grade Teacher
Joan Tederous	First Grade Teacher
Marjorie McCaig	Second Grade Teacher
Genevieve Ronan	Second Grade Teacher

Pictorial Record

Richard W. Hallberg	Jamestown Post Journal
Lamar Schnur	Dunkirk Evening Observer

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Silly Sylvester

Preface

'This Doesn't Seem Like Math ... This Is Fun!'

Six-year-old Marc was very suspicious.

"Are you sure this is math?" he demanded of his teacher.

She nodded gravely. "Yes, I'm sure, Marc." Then, with a smile she asked, "Are you surprised?"

"Wow--I sure am," Marc said. "This doesn't seem like math--this is fun!"

For parents who grew up being made to feel woefully inadequate because they could not memorize their nine-times-tables or tolerate the tedium of trying to remember how many pints in a quart--the experimental math program being piloted at St. Mary's School, Dunkirk, and the five elementary schools in Falconer, New York, may be a bit confusing.

But the enthusiasm of the teachers and children is unmistakable.

No longer is math being thought of as a "subject." The Wisconsin Research and Development Center for Cognitive Learning at the University of Wisconsin encourages children to "think of mathematics as a natural means of expression and communication."

The DMP (Developing Mathematical Processes) program does just that, by utilizing an activities approach to math that children can naturally absorb and enjoy.

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The emphasis is on manipulative materials that children can use to demonstrate for themselves certain mathematical principles. The typical DMP learning kit contains blocks, chain links, scales, balances, cups, coins, dice, yarn, rubber bands, funnels, sugar cubes, toothpicks, weights--even packets of Kool-Aid.

DMP uses a "child's viewpoint to teach math," according to Dr. William Schall, Fredonia State College, who is the New York State Coordinator for the national field testing of DMP.

"Independent exploration is stressed," Dr. Schall said, "and the child sees for himself how to equalize weights or lengths or volumes by adding and subtracting."

St. Mary's and Falconer elementary schools are the only schools in Western New York field testing the revolutionary approach. Because DMP is still in its developmental stages, only four levels (K-2) are available now, but eventually the program will encompass all 6 elementary grades.

Because children tend to view objects comparatively--which is longer, which is heavier, which is bigger--it is possible for them to discover certain mathematical concepts for themselves. DMP is based on this self-discovery. Instead of monotonously writing sums and equations, children learn by doing.

A typical class period may be spent pouring liquids into containers of different sizes to discover which are equal. Or a game

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situation may be set up using dominoes or chain links to teach children how much must be added or subtracted from one number to make it equal another.

Many teachers in the IMP program have made comments, such as the following, about the program:

"Children are expected to prove their answers with tangible evidence. This helps them to learn better and they think it's fun."

"It's not so much drill--they are practicing the concepts with a reason for doing it."

"Children have been too stereotyped until now, led to believe that arithmetic should be counting, number, adding and subtracting. We weren't manipulating objects and materials as we are now."

"IMP offers an activity approach which is the way children learn best--working first with concrete rather than the abstract."

Another attractive feature of the program is that it requires very little reading--so the slow reader does not automatically become a slow math learner, as has often been the case previously. The cartoon characters who "learn" math along with the children delight the students and reinforce their positive reactions to learning arithmetic.

Although IMP is obviously fun for the students, it is nevertheless a strictly controlled program according to Dr. Schall. Children are pre-tested and then evaluated constantly as they proceed. Report cards will not have the traditional letter grades but

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will describe in detail what a child has mastered and what he still needs to learn.

Whatever else it accomplishes, DMP is changing the entire image of arithmetic. No longer is it just dull drudgery, something to be dispensed with quickly or avoided altogether whenever possible.

DMP has students asking, "When do we get to do math?"

Silly Sylvester

Overview of Developing Mathematical Processes (DMP)

During the 1972-73 school year approximately 60 schools participated in the large-scale field testing of DMP Levels 1-4. The schools involved were located in 17 states and in a variety of settings--urban, suburban and non-urban. There were both public and private schools; about one third of the schools were multiunit schools. The size of the schools varied greatly as did the number of DMP teachers in a school.

Other levels of the program are scheduled for testing in 1973-74, 1974-75 and 1975-76. After the field tests are completed, DMP materials will be commercially available from Rand-McNally and Company.

The following quotations explain the activity approach used in the DMP program. They discuss the main characteristics of the program and the principles on which these characteristics are based. The three major characteristics of DMP are (Harvey, et al., 1970):

> The major innovation that is used in the DMP program is the inclusion of geometric ideas at all levels of instruction. The geometry is not the formal geometry studied in tenth grade; rather it is an informal, intuitive look at size, shape, and incidence relationships of two- and three-dimensional objects. A serious attempt is made in the instructional materials to integrate geometry with the study of arithmetic.
>
> This integration is possible because of a second characteristic of DMP: arithmetic is developed from a measurement approach. In a measurement approach, the children themselves generate the numbers they work with. The attributes of length, weight, numerousness, volume, area, time, and money are explored in depth, and the children measure objects, sets of objects, distances, and events so that these entities can be compared and described. The numerical measures are then processed by the children. Because some of these entities are characterized by direction as well as size, the study of positive and negative integers is begun rather early, at approximately third-grade

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level. Because children are constantly generating numerical data, it is considered appropriate to study certain elementary notions of probability and statistics so that data can be organized and analyzed.

A third characteristic of DMP is that the instructional materials are intended for use in a framework of individually guided education (IGE). IGE emphasizes assessment and evaluation of each child's progress and needs. A child's instructional program is designed around a variety of learning situations. For example, materials can be used for independent study, or they can be used with small groups so that the children learn from each other and from the materials.

These characteristics of DMP program are derived from the following principles (Harvey, et al., 1972):

> The main characteristics of DMP's activity approach are based on sound psychological principles that research has shown are important to learning math. These characteristics include having children work individually or in small groups, as well as in large groups, while the teacher acts as a resource person, not a lecturer. And in an activity approach the children use physical materials (Unifix cubes, Lots-a-Links, games, etc.) to help make abstract mathematical ideas more concrete. Also, children work together, discussing the problems they are solving and justifying their answers. Finally, the overall objectives and activities for classroom work are selected by the teacher, but informal activities which result from the interests of the students are pursued by the class when appropriate.
>
> By now it should be clear that an activity approach to math is rather different from that usually found in traditional classrooms. It should be clear, too, that activity-centered math is not turning children loose to riot; nor is it hit-or-miss random learning, with a haphazardly conducted instructional program. In fact, just the opposite is true. DMP's activities are organized and sequenced with great care, so that skills needed at a certain point have already been mastered in prior activities.

In the DMP program children are allowed to work at their own pace until they have mastered a particular topic. If a child seems to be lagging behind, the teacher and the child may retrace some steps in

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the program and plan a little differently for the child. In DMP the sequence of mathematical skills is important, but the needs of the learner are more important. The sequence for each child is paced for him; the child is not paced for the sequence.

In no DMP classroom will all children be on page 37, nor will children necessarily be using their workbooks everyday. Further, children will, usually, not be sitting quietly in neat rows of desks. Often children will be found buzzing around all over the room working with many kinds of materials. An activity approach to mathematics requires that learners be active participators in the learning process. This is the heart of the DMP program.

The following paragraph from the Assessment and Managing Instruction (Harvey, et al., 1972) summarizes DMP's relation to the IGE Model:

> DMP is a complete mathematics program that is designed to help the teacher provide individually guided education for each child. DMP is built on a foundation of carefully specified and hierarchically sequenced behavioral objectives. The children's level of mastery of each objective can be determined by using the DMP assessment instruments which help the teacher decide how to group the students so that each child is working on a set of objectives that are appropriate for him. For each set of objectives there are various instructional activities which teachers can use. In these activities children may work in large groups, in small groups, in pairs, or individually. Activities may involve physical objects, pictures, games, or stories in presenting mathematical concepts. The variety of activities in DMP [illegible] a flexible instructional program that [illegible] the students.

In summary, then, DMP [illegible] a research based elementary mathematics program currently under [illegible] the Mathematics Project of the Wisconsin Research and Development Center. DMP provides a complete

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mathematics program for the elementary school, including not only the usual topics in arithmetic, but also an informal, intuitive introduction to major ideas of geometry, probability, and statistics. This program is based on the principle that children learn best in an active environment where they can seek out answers and strategies to problems of personal interest. This active environment stems from the development of arithmetic through a measurement approach with the children themselves generating and working with the numbers and their relationships.

DMP is different for four reasons. First, it has been developed from a child's perspective--not from the perspective of an adult. Young children are naturally active and curious. They want to find out about the things around them and within their own world. But they do not want to be told about those things; they want to interact with the objects of their world through their senses. The DMP program gives children an opportunity to learn about their world while actively investigating and studying the mathematical aspects of their environment. One should not consider mathematics as something that happens only in a mathematics classroom. Rather, one should seek ways in which one can help children relate mathematics to such aspects of a school program as science, social studies, art, and communication skills.

Second, the DMP program is a carefully designed approach to mathematics learning. At the heart of the design is a set of

interrelated behavioral objectives. Each explicitly stated objective defines a learning goal, and earlier objectives lay the groundwork for those that follow. Each topic in DMP teaches toward a specific set of these objectives, and the activities are designed to help the child master the objectives. Mastery of successive behavioral objectives marks the child's progress through the DMP program. His success will depend in part on the teacher's knowledge of the relationships among these objectives and on the teacher's ability to assess the child's progress with them. The teacher materials accompanying this program are designed to help the teacher do this.

Third, the program includes a wide variety of instructional activities that teach toward the objectives. Each activity has been carefully considered, tried out in a variety of schools, and revised one or more times. The activities ask the children to use the problem-solving processes of mathematics. By engaging in such activities, the child is encouraged to explore the mathematical properties of his world and to talk about and record this information. In so doing, he is developing a natural language and symbolism that has meaning to him.

Fourth, DMP has been designed for Individually Guided Education (IGE). Since not all children are interested in the same aspects of their environment, nor do they learn in the same ways or at the same rates, the DMP program is designed for use in an individually guided educational setting. While most of the activities are designed for use with small or large groups of children, some are designed for use by a child or a pair of children. This gives a child the chance

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to interact with ideas embodied in physical, pictorial, or symbolic representations on his own, in the company of a classmate, or within a small group. He can investigate ideas in depth or he can go on to a new idea as he wishes. This also means that the teacher can guide him toward the type of activity that is best suited for him in terms of his development, learning style, and temperament. Extensive assessment materials are provided to assist the teacher in making these decisions.

In the Summer of 1972, the Falconer Elementary Schools (5 buildings, 10 classrooms), Falconer, New York, and St. Mary's School (5 classrooms), Dunkirk, New York, agreed to participate in the large-scale field-testing of DMP in grades K through second. The teachers, teacher aides, and several administrators met in August, 1972, for a two-day training session for DMP at Fredonia State College, conducted by Dr. William E. Schall, DMP State Coordinator.

The program began in most buildings in September, 1972. However, some schools did not receive their full complement of materials until October, 1972.

Dr. Schall supervised the implementation of the program by providing on-site help to DMP teachers on a bi-weekly basis.

Purpose of the Study

The primary purposes of this study were to: (1) field-test the complete DMP program in fifteen local classrooms, (2) determine the

effectiveness of DMP in terms of teacher attitudes toward the program, and (3) determine the extent to which children attained the objectives established by the program.

Secondary purposes were to: (1) continue to support local schools in the investigation and implementation of individualized programs of instruction, (2) enhance local interest in new and innovative programs in elementary school mathematics and (3) continue a well established program of in-service education for local school personnel.

Peep

Goofy Gilbert

Martin the Munch

Frantic Franny

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Findings

Data were collected to ascertain teachers' attitudes toward DMP, and also, pre- and post-measures of pupils' achievements on the sequence of DMP topics (see Appendix) were administered at the beginning and end of the school year. The same pupil achievement measures were administered to some control classes in selected Falconer buildings. The control classes were selected on the basis of teachers' volunteering to have their classes pre- and post-tested as a comparison sample.

Teachers' Attitudes (Early Measure)

This survey of teachers' attitudes toward the program was administered after the program was in operation for approximately two months. Questions and answers are as follows:

1. Do you spend more or less time on mathematics this year than you did last? How do you feel about this?

   St. Mary's: All five teachers spend more time this year. Four teachers feel that the time is well spent, and one teacher is concerned because she seems to be concentrating on reading and math and excluding social studies and science.

   Falconer: Seven teachers spend more time this year, and one spends the same amount of time. Three teachers like spending more time and consider it worthwhile because the children are interested. Two teachers feel they do not get all of their other work done because of DMP. Two teachers comment that they would like to be able to have more flexible schedules with DMP. One teacher simply thinks it's "ok" to spend more time.

2. Do you spend more or less time preparing mathematics lessons this year with the DMP program than you did last year with your former program? If you are spending more time in preparation this year, explain why you think you do this. Do you spend more time because you choose to, or do you find more preparation time is necessary with the DMP Program?

   St. Mary's: All five teachers spend more time preparing lessons because the program requires more preparation time. They list the following reasons why they need more time: DMP is a new program; the manipulatives take time to set up; and the concepts and vocabulary are new.

Falconer: Eight teachers spend more time preparing lessons, and two teachers have nothing to compare DMP to as they did not teach last year. All of the teachers feel that a lot of time is necessary in preparation. They list the following reasons why they need more time: DMP is a new program; and the materials take time to set up and put away.

How do you feel about this?

St. Mary's: Four teachers feel the time is well spent as they see the results of their lessons. One teacher simply says she feels the program takes more time this year but that it will be a breeze next year.

Falconer: Three teachers feel that the time is well spent as they see the results of their lessons. One teacher responds that she likes to spend more time. Three teachers find that the extra time spent tires or frustrates them. Two teachers resent spending more time. One of them has other subjects to prepare for; and one is spending time she considers unnecessarily spent as she searches for materials not included in the kit.

3. Are your students more interested in math this year than students in former years have been? How do you feel about this?

Frantic Franny

St. Mary's: Four teachers think their students are more interested in math and one does not. The four teachers of interested students are pleased. The other teacher thinks her pupils do not realize math as a subject yet.

Falconer: Eight teachers respond that their pupils are more interested, and one teacher says she isn't sure. Six teachers are pleased that their pupils are more interested. Two teachers think their pupils may be more interested in the materials rather than the math. The one teacher who is unsure says she's never had trouble getting pupils interested in other programs.

4. Do you like teaching the DMP math program more than you liked teaching the other programs you have taught? Why?

St. Mary's: Four teachers like DMP better; one is unsure. The teachers who like the program better like it because it places less emphasis on computation and textbook math and more emphasis on pupil involvement and discovery with different activities and objects. The teacher who is unsure of her reactions to DMP wants to reserve judgment on this question until the end of the year.

Falconer: Five teachers like DMP better; three teachers don't necessarily like it better. The teachers who like DMP better say that they do prefer DMP because their children respond to and enjoy an activity approach more. They also

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like the fact that their pupils seem to understand math concepts better. Two teachers prefer DMP because they had no formal math programs in kindergarten previously, and DMP gives them a sense of direction and continuity that their informal programs did not. Two of the three teachers who don't necessarily like DMP better than their previous program say that they have always approached math in an activity-oriented way, and so they notice little difference with DMP. One teacher who doesn't prefer DMP doesn't like to follow the book as closely as she feels is necessary in DMP.

5. Do you find space or the physical limitations of your room a problem with the DMP Program?

St. Mary's: One teacher does find space a problem; one teacher finds it a problem sometimes; and three teachers do not find space a problem. The two teachers who think they need more space need display and/or storage space so that their children can have easy access to the DMP material.
Falconer: Four teachers find space a problem and six do not. The four teachers who think space is a problem would like to have more storage and display space for DMP materials.

6. React to the activities and grouping procedures you are asked to use in the DMP Program.

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St. Mary's: In general the teachers liked the activities and grouping procedures. They say the activities are exciting and that they like the enthusiasm with which the children work on their own. One teacher comments that activities and grouping procedures have complicated directions. Another dislikes the noise that occurs during these activities but admits it is constructive. Another comments that some activities seem worthless. Two people feel that the small groups used in the program need more adult supervision than is available.

Falconer: Some teachers respond very favorably to the activities and grouping procedures. They think their pupils are highly interested in the procedures, and that they work well with them because the directions are so easy. They feel that the pupils are learning how to work in a group and that the group activities provide opportunities for all children to contribute. They mention that they like the flexibility DMP allows them as they work with groups.

Other teachers make some unfavorable remarks concerning activities and grouping procedures. They comment that it is hard to work with the groups unless one does have an aide or other help. They say that the preparation time required to prepare for the activities and/or groups and to put away after them is too long. One teacher says she has trouble

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working with groups because different types of groups require varying amounts of time, and thus she has scheduling problems. Other teachers say that some grouping procedures repeat too much and that some have not worked out too well. Some teachers find that noise is a problem with these procedures. One teacher comments that she finds that materials get lost as they are scattered over the room. Then she's short of materials on some other activities.

7. Do you find the DMP teachers' manual adequate?

St. Mary's: All five teachers think the manual is adequate. One comments that it includes everything including problems you may encounter.

Falconer: Nine teachers think the manual is adequate. One teacher finds the manual hard to understand. One comments that instructions concerning the workbooks have not always been adequate.

List specific page numbers and questions you have had concerning specific topics and/or activities. (If the manual has not been clear, list the page number here and state what has not been clear.)

St. Mary's: Level 2, Activity 3.5--the ditto concerning mine, yours, and ours is hard to explain. Level 3, page 19--the objects should be labeled on the inside. Level 3, page 61--the manipulative materials are not adequately described. (Example: geometric materials sets R and S).

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Peep

Falconer: Level 1, Introduction--this activity has no similarities except possibly eyes. It is very confusing to the children at this point. They naturally look for similarities rather than differences. Level 2, page 98--part A--the teacher thinks the containers for this lesson should be in the kit. Level 2, activity 3.5--this activity is much too difficult for children to do without previous explanation. Level 2, page 111, activity B--weighing an object that weighs one or two larger washers would require too many lots-a-links. There isn't room in the cup of the balance beam for them all. Level 2, page 135--optional activity 2.5 10 Experiment one is too messy for the average classroom. Level 3, Introduction--there is a printing error here. "The equalizing process can be divided out by adding on to the larger or taking away from the smaller." Level 3, page 35--A, C, and A should say squares instead of boxes because you need squares on the next page. Level 3, page 43--Topic 3.2, objective one. Figures should be labeled and defined for teachers. Level 3, page 61--the manipulative material descriptions are not always adequate. (Example: geometric shapes sets R and S) Level 3, page 88--Topic 3.3.1, objective 4. This activity validates sentences greater than 10. Should it? Level 3, page 96--Topic 3.3.3, activity A. The counting chips tend to all go into one pocket. We had many problems with a zero

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addend. Level 3, page 121--Topic 3.3.9 Teachers need a copy of the story for level 3.9.8. Level 3, page 141--Topic 3.4.2. Story problems A-H are missing. Level 3, page 145--Topic 3.4.3 Activity B used 15-20 counters instead of 20-40. Level 3, page 162--two answers are wrong on this page.

What are your reactions to the pupil workbooks?

St. Mary's: Three teachers reacted favorably to the workbooks. They think the workbook pages are a good culmination activity to the physical preparatory experiences their children have had. One teacher comments that she thinks it is important for her pupils to work with pictorial objects as well as physical ones. Two teachers do not especially like the workbooks. One says that there are too many pages for each lesson in the workbooks. She also comments that the pages are too crowded and the pictures (especially the ones that are supposed to aid the children in counting) are too small. (An example of these small pictures is attached at the end of this survey. It is from a pupil workbook for level three.) This person also thinks that the numbers used on the workbook pages are too small. Level two workbooks contain small numbers on pages 17, 18, and 32. Another teacher comments that the directions for the pupil workbooks are

hard to follow and that the organization of them is unclear. She points out that the numbers in the level three workbook on page 19 should be on the pictures rather than next to them. After the pictures have been cut out, they cannot be identified.

Falconer: Most teachers reacted favorably to the workbooks. Some comment that they like the perforated pages. One teacher likes the black-and-white pictures. Several teachers mention that the workbooks provide individual activities for the students that help to identify each child's level of mastery of the concepts. The teachers make the following comments about specific parts of the workbooks. Level 1--there are more pages per lesson than kindergarten children can do. All of the pages take too much time. On pages 13 and 14 why do Mr. Long and Mr. Short wear dresses? On pages 17, 19, and 26 pupils have trouble figuring out which pictures to cut out to complete the patterns. On page 26 the only boy with a worm catches a fish. The length of the line is secondary. Level 2--pages 45-47 should be rearranged. Level 3--pages 59 and 60 are well done. Pages 63-69 the counting activities are difficult.

8. Does the assessment manual adequately explain the assessment procedures used in this program? List your concerns, problems, questions, and reactions to the assessment program here.

Glump

St. Mary's: All five teachers think the manual adequately explains the assessment in the DMP Program. One teacher comments that it is difficult to assess children when activities are being done by other children. One teacher thinks the assessments help her know each child's level of mastery better. A kindergarten teacher says that she finds her children tend not to gain mastery of task 3 in topic inventory 2 when she uses five pencils of different lengths as the task calls for. However, she finds that the children do obtain mastery when she uses larger objects of different lengths in the task.

Falconer: Eight teachers think the manual adequately explains the assessment program. Two teachers did not answer this question. Two teachers comment that they like the assessment program because they feel they cannot determine which children have gained mastery on a task via informal observations. The assessments provide them with objective evidence. However, one of these teachers comments that shy children tend to perform badly on the assessments when they are individually administered so she tries to assess shy children in different ways to determine their progress. Another teacher comments that the assessment manual is confusing when it discusses methods of recording levels of mastery. Two other teachers say that the assessment program is too easy. One comments

that some tests such as Topic Inventory 2.4 do not test all of the objectives that have been covered in that topic. One teacher finds that the assessment program is very time-consuming. Another teacher thinks the assessment manual should be printed in the teacher's manual in red ink rather than as a separate manual. She thinks it is difficult to constantly use the two manuals.

What are your reactions to the pupil test books?

St. Mary's: The teachers basically find the booklets helpful. One comments that she prefers to assess progress with a written record rather than an observation. She thinks the booklets adequately assess retention of concepts. One teacher comments that the booklets are easy to correct. One thinks that the booklets need some color.

Falconer: Two teachers think that the test booklets are too easy. Two think they are fine and one comments that they are adequate. One of these teachers comments further that the test books are valid in her opinion and make it easy to determine if pupils are meeting the objectives. A kindergarten teacher comments that the Placement Inventory was long and difficult to administer; it frustrated her children because it took so much time. A second-grade teacher commented that she has used only two topic inventories. She

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finds that the results of these tests correlate closely with class performance of the children. One teacher says that page 18 in the level two manual contains a mistake. She says one sentence should read "The set of dolls is larger than the set of bats." The sentence now reads "The set of bats is larger than the set of dolls."

9. Does the manipulative materials kit contain enough materials for your classroom use? What do you need more of, if anything? How many students do you have in your class?

St. Mary's: One teacher who has 23 students says she has enough materials. The other four teachers who have 15, 24, 19, and 21 students respectively do not have enough materials. All four teachers need more geometric solids. The teacher with 24 students needs more lots-a-links and unifix cubes. The teacher with 21 pupils needs more geoboards and geometric pieces.

Falconer: Five teachers with 19, 28, 22, 20, and 21 pupils respectively have enough materials for their respective groups. One teacher who does not have enough materials doesn't say how many pupils she has. Two teachers who have 23 and 22 pupils respectively comment that they usually have enough materials. The teachers who need more materials list their needs as follows:

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Symbol Simon

Two teachers with 22 and 23 pupils need more geometric pieces. Five teachers with 20-23 pupils need more balance beams. One teacher with 22 students needs more colored counting chips. One teacher comments that all plastic jars and containers required in DMP should be in the kit. Another teacher says that her kit did not contain all the items it should have. (Several containers, an adding machine tape, and a package of rubber bands were missing.) Another teacher comments that she would like to see the geoboards made of materials other than plastic.

Does the materials kit contain items which you find are not used? List these items.

St. Mary's and Falconer: The teachers are unable to answer this question at this time because they haven't completed reading their manuals. They won't know until near the end of the year if they use all the materials. One teacher does say that she hasn't found a use for the one-pound weight or the 3"-wide plain paper yet (Level 3).

10. Do you have a teacher aide, student teacher or other help during the DMP Math Program? How does this person assist you?

St. Mary's: Four teachers have a teacher aide all of the time, and one teacher has one sometimes. Four teachers report

that their aide works with groups. One aide helps with assessment procedures. She administers all of the topic inventories. One aide gives individual help to children as they need it. Another aide helps the children in secondary ways by clarifying directions and passing out material. One teacher uses her aide very little in DMP.

Falconer: Ten teachers have a teacher aide. Two teachers had a student teacher in addition. The student teachers actually taught the DMP lessons. Seven of the teacher aides work with individual pupils; four aides assist with group work; two work with assessment; four aides help prepare and/or set up manipulative materials. One aide assumes the role of an adult leader when such leaders are required. Two aides check children's workbooks and tests, and one aide tears out workbook pages as she helps prepare for the lesson. Three teachers comment that they think aides should have their own manuals.

11. Comment here about consulting services. Have you gotten the help you really need?

St. Mary's: All five teachers say they have received help when needed. They comment that their questions have always been answered and that they've always been able to get advice when needed. They like the frequent visits made by the coordinator.

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Falconer: Eight teachers state that they have received all the help they have needed. Two teachers did not answer the question directly. Both teachers would have liked more help from other DMP teachers. They feel that this type of service has not been provided. Another teacher states that she could have used more help in the beginning when DMP was totally new. As she has become used to the program, consultant service has been fine. Many teachers expressed that the help they've received from Dr. Schall has built up their confidence in DMP.

Additional Comments:

One teacher says "DMP has been an interesting challenge. I thoroughly enjoy the activity program. I like to see everyone taking part."

Another teacher says, "I will comment further after I have been involved with DMP a full year."

A third teacher responds, "Because I am a new teacher and have not taught math before, I cannot make a comparison between DMP and other math programs. However, I feel very positive about the program for kindergarten children. With the use of manipulations, children have learned to classify, to describe, to compare, to order, to state attributes;

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to do many things they would be unable to do without a program of this sort. The program seems based on problem solving rather than rote learning. In such a program children learn how to learn rather than memorizing facts. I am curious to know what the retention will be when the student reaches grade one."

Teachers' Attitudes (End-of-Year Measure)

The Teachers' Attitudes Instrument was administered at the end of the school year. Scaled responses and open-end comments from the DMP teachers were ascertained using the following 3-point rating scale:

1 = if it was poor or unsatisfactory

2 = if it was average (acceptable)

3 = if it was excellent or outstanding, extremely good.

	3	2	1
1. The DMP is a carefully sequenced mathematics instruction program.	16	-	-
2. The inclusion of geometric ideas in the DMP is an educationally sound innovation.	15	1	-
3. Each student's contribution in generating numbers helps the child in his learning.	13	1	-

"Group does not contribute because of lack of experiences."

"Not to mastery for all students--but have excellent results."

	3	2	1
4. The program offers ample opportunities to students to draw on their own experiences. "Some students don't take advantage of these."	13	3	-
5. The DMP materials provide a variety of learning situations for a concept or a principle. "This variety picks up the students who might get the concept if approached another way."	16	-	-
6. The assessment and evaluation of each child's progress and needs is made possible through the use of the curriculum package. "This is one of the strongest points of the program to me. If these three are easy to keep track of, each child's needs are more easily planned for with a time shortage in mind." "Some of the assessments were too easy." "I feel that the testing was too easy sometimes."	16	-	-
7. The curriculum package, teacher's guide, pupil texts, physical manipulative aids, and assessment materials are based on sound learning and teaching principles.	16	-	-

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8. Compared to the other subjects you teach, you enjoy teaching DMP (check one)

more than any other	better than most	moderately well	less than most	least of all
3	9	3	___	___

"as much as" 1

	YES	NO
9. Now that you have had the experience of teaching with DMP for one year, would you recommend the use of the program again next year?	16	0
10. Based on your experience of teaching with DMP for one year, would you recommend the use of the program to your colleagues? (check one)	16	0
11. You spend more time on DMP this year than you did last year and you feel the time is well spent. (check one)	15	1

12. This year, it was possible for you to individualize instruction using the DMP materials: (check one)

all of the time	much of the time	some of the time	never or rarely
___	7	9	___

"The program itself provides for individual differences."

"Due to inexperience I have kept the group pretty much together--individualizing within the same lesson by varying activities."

"Need assistance to do this."

"I feel that most of individualization was due to the help of our aid."

"Would do more if possible."

"Very good--many ideas for alternatives, good instructions."

"Very good, lots of alternatives--suggestions."

"Protective cover needed."

"I'd like to see a hard cover on the Teacher's Guide."

"Laminate cover."

"Teacher's Guide is quite comprehensive."

"Very good."

"OK."

"Some lessons too long for one day. Teachers must realize these lessons are to be spread over 2 or more days."

"Excellent."

"Many mistakes were found which I have passed on to coordinator. Identification of materials not clear--geometric pieces 'set r&s' was unclear. Why not call them set 'bg' (blue-green) and 'ry' (red-yellow)?"

"Just that there has been some misprints."

"I think there should be a key for the workbook pages and that the workbook pages' directions be better explained. I think DMP could provide some of the dittoed sheets we have to make, in the workbook."

Symbol Simon

c. Pupil Texts

"Complete enough, dittos should be included."

"Fine."

"Curriculum pkg. some stories are too long. K pupils cannot listen that long (1.13.1 for example)."

"Not enough graph paper for lessons supplied."

"Fine."

"I lack any sort of graph paper for the pupils even though many lessons called for it."

"I wish these were not individual books but that there were stacks of each page."

"OK."

"Workbooks--some pages were too cluttered."

"Adequate."

"Favorable."

"Add more color and use more space."

d. Physical Manipulative Aids

"Enough."

"Fine."

"Very good."

"Good."

"Need extra set of geometric forms."

"Some odd-shaped containers which are difficult to find should be included. Rubber bands that are stronger should be used."

"Stronger rubber bands, some type of container--sectioned for storage of small plastic pieces, etc."

"Very good. Children enjoyed working with materials."

"Some material was worthless and other not included. Using string or yarn for paths and location was hard for them."

"Kit should provide some of the odd containers that you can't find. Stronger rubber bands and the source for supplying them. Provide Rand McNally Catalog."

"Unique."

"Favorable."

"Are very good."

"More cubes and links and solids should be packed--it is not necessary to send so many rubber bands, for instance."

e. Assessment Materials

"Fine."

"Fine."

"Very good, but more ways of assessing than necessary."

"Good."

"Some assessments seem to be quite easy."

"Fine."

"Good for the most part. Most are easily supplied."

"I did think the assessments were a little too easy. I feel that I gave only complaints. I like DMP and my children do very much."

"OK."

"Sufficient."

"Good--quick and easy. Also, various phases are broken down so that it is easy to pin-point problems."

"There have been some misprints in the group-administered tests."

f. Storage

"I bought a parts cabinet to store small geometric shapes, dice, rubber bands, etc. It is a big help."

Pupil Achievement

Table 1 shows two types of scores: (1) the median scores on the DMP inventories indicate the extent to which pupils demonstrated mastery of the sequence of topics offered in the DMP program; and (2) the mean grade scores show a normative measure of children's responses to standardized achievement tests. Falconer used the Stanford Achievement Test; St. Mary's used the Metropolitan Achievement Test. Both achievement tests were administered at the end of the year.

Table 1

Median Scores on DMP Inventories and
Mean Scores on SAT and MAT

School	Falconer																	St. Mary's				
	K Experimental				1 Experimental			1 Control			2 Experimental			2 Control				K Ex.	1 Ex.		2 Ex.	
Test	1	2	3	4	1	2	3	1	2	3	1	2	3	1	2	3	4	1	1	2	1	2
Number of Children	23	18	42	54	21	22	22	25	24	23	22	20	21	19	14	18	24	29	24	22	21	20
*Inventory A (Pre)	-	-	14	-	20	20	22	21	21	18	-	-	-	-	-	-	-	15	22	21	-	-
Inventory A (Post)	22	19	19	22	22	24	-	23	23	22	-	-	-	-	-	-	-	21	23	21	-	-
Inventory B (Pre)	-	-	-	-	-	-	-	-	-	-	-	-	-	-	-	-	-	-	-	-	18	-
Inventory B (Post)	-	-	-	-	26	26	27	23	22	29	-	-	-	-	-	-	-	-	28	27	-	-
Inventory C (Pre)	-	-	-	-	-	-	-	-	-	-	22	18	30	21	17	43	24	-	-	-	-	25
Inventory C (Post)	-	-	-	-	-	-	-	-	-	-	50	49	52	40	-	49	48	-	-	-	53	52
SAT	-	-	-	-	1.9	1.6	1.8	2.4	2.2	2.3	2.8	2.3	2.9	2.8	-	2.9	2.9	-	-	-	-	-
MAT	-	-	-	-	-	-	-	-	-	-	-	-	-	-	-	-	-	-	1.6	1.6	4.2	3.4

*Inventory A has 24 items; Inventory B, 34 items; and Inventory C, 54 items.

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Discussion

An analysis of results indicates that the DMP program may be said to have achieved a considerable measure of support from teachers who made use of DMP material in their classes. In fact, teachers' attitudes toward DMP were highly favorable. Their suggestions for further improvement of DMP materials are extremely constructive and will be incorporated by the developers of DMP materials.

Criterion-referenced DMP measures (Inventories A, B, and C) indicated that both DMP children and control children demonstrated substantial growth in terms of attaining mastery of the mathematical concepts.

Differences between Falconer DMP and control* children on median pre- and post-test scores were as follows: (1) no comparisons are available for kindergartens because there were no control kindergarten classes; (2) first grades gained equally; and (3) second-grade DMP classes gained substantially more than did the control classes.

With reference to standardized achievement scores, the data showed that both first- and second-grade control classes scored slightly higher than did DMP first- and second-grade classes on computations and that control classes scored somewhat higher on math concepts.

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* It should be noted that neither the DMP group nor the control group were representative of their grade levels. Both of the groups were comprised of children whose teachers volunteered to participate in the study. Control children were from the alternate buildings located in this small urban area. Most DMP classes were from the more rural disadvantaged area of the Falconer Central School District; the remaining DMP children were from a small Catholic school in Dunkirk, New York.

It should, again, be emphasized that DMP approaches mathematics by developing processes (which must consume much of the time traditional programs spend on computation and practice/drill) which enable children to solve problems, one of the most important goals of mathematical education.* It is upon these processes and attributes that the program is criterion referenced or evaluated. Children in DMP first directly examine attributes and processes and then, in progression, represent these attributes and processes physically, pictorially, and symbolicly. Both this progression and the development of the process of validating are included in this initial part of the program (levels 1-4). Of course, not all mathematical problems are arithmetic in nature; DMP also considers problems from geometry, statistics and probability. These subject problem/processes areas are treated under the "describing and classifying category" in the program.

Inasmuch as DMP is sequenced over a different continuum (processes and attributes), the standardized achievement test scores represent an unfair and invalid measure of mathematical success as taught through DMP. Most of the items that comprise standardized tests are designed to measure the scope and sequence of the more traditional mathematics programs. DMP does not include much of the computational skills and/or concepts until later levels. Therefore, one must, indeed, be careful of any conclusions and/or extrapolations drawn from standardized test data at this time.

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* Developing Mathematical Processes, Scope and Sequence Chart, Wisconsin Research and Development Center for Cognitive Learning, 1972, page 1.

Silly Sylvester

References

Harvey, John G., Moser, James M., and Romberg, Thomas A. Developing mathematical processes sampler, level two. Madison, Wisconsin: Wisconsin Research and Development Center for Cognitive Learning. The University of Wisconsin, 1970, pp. 1 and 2.

Harvey, John G., Moser, James M., and Romberg, Thomas A. An activity approach to math. Madison, Wisconsin: Wisconsin Research and Development Center for Cognitive Learning. The University of Wisconsin, 1972, p. 1.

Harvey, John G., Moser, James M., and Romberg, Thomas A. Assessment and managing instruction. Madison, Wisconsin: Wisconsin Research and Development Center for Cognitive Learning. The University of Wisconsin, 1972, p. 3.

Appendix

Developing Mathematical Processes

Scope and Sequence Chart

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DEVELOPING MATHEMATICAL PROCESSES

Developing Mathematical Processes (DMP) approaches mathematics by developing processes which enable children to solve problems, one of the most important goals of mathematics education. It is upon these processes and the attributes with which they are connected that this scope and sequence chart is based. The categories specified along the side of the chart are the basic processes of DMP. There are two other important processes, representing and validating, that are used in conjunction with these basic processes. Children in DMP first directly examine attributes and processes and then, in progression, represent physically, pictorially, and symbolicly. Both this progression and the development of the process of validating are indicated within the body of the chart.

To be able to solve mathematical problems, a child needs to understand different number systems and relations and operations on these systems. These processes lend themselves to this development. You will find these familiar subjects under the following process categories.

Subject	Process Category
number systems (whole numbers, fractions, integers)	describing and classifying
order relations	comparing and ordering
addition and subtraction	equalizing, joining and separating
multiplication and division	grouping and partitioning

Not all mathematical problems are arithmetic in nature; DMP also considers problems from geometry, statistics and probability. You will find the main treatment of these subjects under the describing and classifying category.

After each statement in the body of the chart are the numbers of the topics in which the indicated behaviors are emphasized. This does not mean that such behaviors are not introduced earlier and reviewed later; nor does this mean that the behaviors constitute the entire emphasis of those topics.

Note: Topics 55 - 96 (Levels 6 - 8) are being developed and are subject to change on the basis of evaluative data received from field tests. (July, 1973)

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TOPICS 1-14

DEVELOPMENTAL - LEVEL 1
APPROXIMATE GRADE - K

DESCRIBING AND CLASSIFYING	ASSORTED ATTRIBUTES-verbally describes objects' likenesses and differences; sorts objects into sets on obvious visual attributes; makes patterns (1) LENGTH-physically represents a length of an object by making a chain or a string as long as the length; pictorially represents a length by graphing (5,11) NUMEROUSNESS-physically represents a set by matching objects with members of the set; pictorially represents a set by graphing or tallying; recognizes numbers 0-10; counts 0-10; symbolically represents a set with a number 0-10 (9,11,12,14) TIME-uses words such as morning, afternoon, evening, day, night, spring, summer, fall, winter to tell when an event occurred (13) MONEY-recognizes and counts pennies, ¢ sign introduced (14) 3-D SHAPE-describes shape of solids in terms of roundness, flatness, height, ability to stack, roll or slide; sorts (8) MOVEMENT-verbally describes the direction of a movement; physically represents a movement with a string path; pictorially represents a movement on a map (6) LOCATION-verbally describes location of an object using another object or a path as a reference; uses words such as above, below, on, inside, outside; gives simple directions (10)
COMPARING AND ORDERING	LENGTH-directly compares and orders lengths; uses physical or pictorial representations to compare and order lengths (2,4, 5) NUMEROUSNESS-compares and orders sets by matching one-to-one; matches physical or pictorial representations one-to-one to compare and order (7) TIME-tells whether an event happened before, after, or at the same time as another; decides which of two events, that began at the same time, lasted longer (13)
EQUALIZING	LENGTH-makes two lengths or representations of length equal by adding on to the smaller or taking away from the larger (3) NUMEROUSNESS-makes two sets or representations of sets equal by adding on to the smaller or taking away from the larger (7)

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TOPICS 15-27

DEVELOPMENTAL - LEVEL 2 AND
TOPICS 3.1 AND 3.3
APPROXIMATE GRADE - 1

DESCRIBING AND CLASSIFYING	LENGTH-symbolically represents lengths with a number and arbitrary unit after making a physical representation and counting the units; i.e., measures graphs (19,21,23) NUMEROUSNESS-counts 0-20; recognizes numbers 11-20; writes a number to represent a set 0-20 (17, 24) WEIGHT-measures using arbitrary units; graphs, weights (19,21,23) CAPACITY-symbolically represents with a number and a liquid or solid arbitrary unit after making a physical representation and counting the units (22) MONEY - nickels and dimes are introduced (24) MOVEMENT-pictorially represents movement on a map; symbolically represents movement by listing in order points passed through; makes a movement given directions; uses words such as right, left, up, down, forward, backward (26) LOCATION-verbally describes location of objects using paths as a reference; uses the words right and left; gives directions (26) 3-D SHAPE-physically and pictorially represents faces of solids to obtain 2-D figures; counts faces (15) 2-D SHAPE-counts sides and corners of figures; makes figures on a geoboard; looks at attributes of square, rectangle, and triangle (15,20)
COMPARING AND ORDERING	LENGTH-compares and orders two or more lengths by measuring each with arbitrary units (19) NUMEROUSNESS 0-20-matches two or more sets one-to-one or counts to compare and order; writes order sentences (17,19,24) WEIGHT-directly compares and orders using a balance beam; compares and orders two or more weights by measuring each with arbitrary units (19) CAPACITIES-directly compares and orders by pouring from one container to another; compares and orders two or more capacities by measuring each with arbitrary units (22) AREA-directly compares and orders two areas visually or by superposition(21,23) TIME - orders durations of two or more events which began at the same time; orders times of occurrence for two or more events (18)

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COMPARING AND ORDERING	SENTENCES 0-10-writes and reads comparison sentences such as 9 = 9 or A ≠ B and order sentences such as C < D, 3 < 8, 4 = 4; chooses a sentence which represents a given situation; solves open comparison and order sentences such as □ < 5; validates comparison and order sentences; these sentences are about many attributes: length, numerousness, weights, area, capacity, and money (21,23)
EQUALIZING	WEIGHT-makes two weights equal by adding on to the smaller or taking away from the larger (16) SENTENCES 0-10-writes and reads open equalization sentences such as 5 + □ = 9 or A = D - □ to describe how two sets, lengths, weights, or areas can be equalized; solves open sentences by using objects or pictures; chooses an open sentence which represents a given situation; validates equalization sentences which have been solved (25,27)
JOINING AND SEPARATING	LENGTH-puts two lengths together or takes away from a single length (19,24) NUMEROUSNESS-puts two sets together or takes away from a single set (19,24) WEIGHT-puts two weights together or takes away from a single weight (19,24)

TOPICS 28-42

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DEVELOPMENTAL- LEVELS 3 & 4
(except Topics 3.1
and 3.3)
APPROXIMATE GRADE - 2

DESCRIBING AND CLASSIFYING	ASSORTED ATTRIBUTES-sorts objects on more than one attribute; makes patterns using more than one attribute (36) LENGTH-measures using standard units of inches, feet, yards, centimeters and meters; graphs lengths (39) NUMEROUSNESS 0-99-groups by 10, writes the numbers in expanded and compact notation; reads numbers; makes a set of a given size (33) WEIGHT-measures using standard units of ounces and pounds; graphs weights (42) CAPACITY-measures using standard units of cups, pints, quarts, and gallons; graphs capacities (34) LOCATION-describes location of an object in reference to another object, one or more paths, or a grid (36) 3-D SHAPE-counts number of faces and parts of a solid; describes shape of faces (28) 2-D SHAPE-folds to decide whether a figure is symmetric; identifies and looks at attributes of triangle, rectangle, square, circle; makes figures on geoboard and on geopaper (28,31,40) FRACTIONAL PARTS-divides areas, lengths and sets into fractional parts; symbols introduced after children are familiar with words; finds fractional parts of areas, lengths, etc. (28,37,39)
COMPARING AND ORDERING	LENGTH-orders two or more by measuring each with standard units; orders two standard measurements such as 8 ft. and 3 yards using rulers or measuring tapes (42) NUMEROUSNESS 0-99-orders sets of objects; puts numbers in order; writes order sentences (32) WEIGHT-orders two or more by measuring each with standard units; orders two standard measurements using balances (46) CAPACITIES-orders two or more by measuring each with standard units; orders two standard measurements such as 5 cups and 2 pints by using containers (34)

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EQUALIZING	SENTENCES 0-20-writes and reads open equalization sentences such as 17 - ☐ = 5; solves open sentences using objects or pictures before symbolic solving is required; validates solutions; these sentences are about many attributes: length, numerousness, weight, capacity and money (35) BASIC FACTS-masters basic addition and subtraction facts (35) SENTENCES 0-99-writes and reads open equalization sentences; solves all kinds of equalization sentences using objects or pictures; solves sentences of the type 28 + 36 = ☐ , 91 - 32 = ☐ using algorithms; validates solution; these sentences are about many attributes (38,41) ALGORITHMS-masters 2-digit algorithms for addition and subtraction with regrouping (41,46)
JOINING AND SEPARATING	SENTENCES 0-20-writes and reads open joining and separating sentences such as 3 + 15 = ☐ or 18 - 6 = ☐ ; solves open sentences using objects or pictures before symbolic solving is required; validates solutions; these sentences are about many attributes: length, numerousness, weight, capacity and money (35,38,39) BASIC FACTS-masters basic addition and subtraction facts (35) SENTENCES 0-99-writes and reads open joining and separation sentences; solves sentences of the type 18 + 43 = ☐ and 59 - 27 = ☐ using algorithms; validates solutions; these sentences are about many attributes (38,41) ALGORITHMS-masters 2-digit addition and subtraction algorithms with regrouping (41)
GROUPING AND PARTITIONING	NUMEROUSNESS-makes groups of a specified size and represents the grouping with grouping notation such as 3(4) + 2 (29) NUMEROUSNESS 0-99-groups by 10, writes and reads expanded and compact notation; place value emphasized (33,38) NUMEROUSNESS-partitions by making a specified number of groups and represents the partitioning with grouping notation such as 7(2) + 0 (37)